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Doubletons and 5D higher spin gauge theory

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Abstract: We use Grassmann even spinor oscillators to construct a bosonic higher spin extension hs$(2,2)$ of the five-dimensional anti-de Sitter algebra SU$(2,2)$, and show that the gauging of hs$(2,2)$ gives rise to a spectrum $\mathcal{S}$ of physical massless fields with spin $s = 0, 2, 4, \ldots$ that is a UIR of hs$(2,2)$. In addition to a master gauge field which contains the massless $s = 2, 4, \ldots$ fields, we construct a scalar master field containing the massless $s = 0$ field, the generalized Weyl tensors and their derivatives. We give the appropriate linearized constraint on this master scalar field, which together with a linearized curvature constraint produces the correct linearized field equations. A crucial step in the construction of the theory is the identification of a central generator K which is eliminated by means of a coset construction. Its charge vanishes in the spectrum $\mathcal{S}$, which is the symmetric product of two spin zero doubletons. We expect our results to pave the way for constructing an interacting theory whose curvature expansion is dual to a CFT based on higher spin currents formed out of free doubletons in the large-N limit. Thus, extending a recent proposal of Sundborg (hep-th/0103247), we conjecture that the hs$(2,2)$ gauge theory describes a truncation of the bosonic massless sector of tensionless type-IIB string theory on $AdS_5 \times S^5$ for large N. This implies AdS/CFT correspondence in a parameter regime where both boundary and bulk theories are perturbative.

Keywords: Field Theories in Higher Dimensions, AdS/CFT Correspondance, Space-Time Symmetries, Supergravity Models.

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Contents

1. Introduction

Motivations for studying higher spin fields have varied in time. To begin with, "they are there", in the sense that there exist higher spin representations of the Poincaré group and therefore it is natural to seek field theories which would describe particles that carry these representations. In fact, already in 1939, Fierz and Pauli [1] studied the field equations for massive higher spin fields. They studied the free field equations, and while potential difficulties in constructing their interactions were recognized, the real difficulties became more transparent much later. The Fierz-Pauli type equations in flat spacetime were developed further in 1974 [2] and their massless limits were obtained in 1978 [3, 4].

Difficulties in constructing the interaction of higher spin fields were better understood by the early eighties, both in the S-matrix [5, 6] and field theoretic [7, 8, 9] approaches. These studies, which led to certain no-go theorems, made certain assumptions though, which turn out to be too restrictive as was discovered later. Among the assumptions made were Lorentz invariance (thus, neglecting the possibility of anti-de Sitter invariance, for example) and the fact that one higher spin field at a time was considered (thus, leaving open the consequences of introducing infinitely many higher spin fields).

Interest in a search for consistent interactions of massless higher spin fields received a boost with the discovery of supergravity in mid-seventies. Among the reasons for the renewed interest in the subject were: a) to better understand the uniqueness of supergravity theory; b) to search for supergravities with higher ($N > 8$) extended supersymmetry which would involve larger Yang-Mills gauge symmetries with better grand unification chances; c) the possibility of a better quantum behaviour by the

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inclusion of higher spin gauge fields; and d) from a purely theoretical point of view, to develop a deeper understanding of gauge theories that goes beyond Yang-Mills and include massless fields of arbitrary spin. Most of the attempts made in these directions, some of which are mentioned briefly above, essentially led to negative results. Nonetheless, an interesting development took place in 1978 and the massless higher spin gauge theory problem was shifted to what appeared to be a more complicated setting, namely anti-de Sitter space. In fact, this shift turned out to be crucial for the subsequent breakthroughs that took place in the development of higher spin gauge theory, as we shall explain briefly below.

In 1978, Flato and Fronsdal [10] established that the symmetric product of two ultra-short representations of the anti-de Sitter group in four dimensions, known as the singletons, yields an infinite tower of higher spin massless representations. Motivated by this result, the free field equation for massless fields of arbitrary spin in AdS_4 were constructed in 1978 [11, 12]. Moreover, it was suggested by Fronsdal [11] that "a theory of interacting singletons will provide an example of interactions between massless fields with higher spins". We will come back to this point later.

Nearly a decade later, in 1987, Fradkin and Vasiliev [13, 14] made an important dent in the problem of interacting higher spin gauge theory. They showed that the gravitational interaction of massless higher spin fields does exist after all, provided that the construction is based on an infinite-dimensional extension of the AdS_4 algebra and that the interaction is expanded around an AdS_4 background. One may argue that the key to this development is the recognition of the importance of a suitable choice of higher spin symmetry algebra. It is intuitively clear that once a generator with spin higher than two is introduced, insisting on a Lie algebra, its closure will require the introduction of an infinite set of higher spin generators as well. To see this, it is sufficient to consider the AdS generators as bilinears of suitably chosen oscillators with natural commutation rules, and the higher spin generators as polynomials of higher than quadratic order in these oscillators. What is more surprising, at least at first sight, is the fact that the flat space limit cannot be taken in the interactions involving higher spin fields. On the other hand, this is how the theory manages to circumvent the no-go theorems of [5, 6] which were based on Minkowskian S-matrix considerations.

Until the late eighties, the massless higher spin gauge theories were mainly being considered in their own right, though undoubtedly a great deal of motivation must have been gathered from the by then well-established conviction that higher spin gauge theories do exist and as such should necessarily have to have a bearing on unified theories involving gravity. Nevertheless, no particular significance was yet attached to the remarkable connection between AdS_4 singletons and massless higher spin fields discovered nearly a decade ago [10]. Moreover, no connection with string theory or any theory of extended objects was made yet despite the fact that the theory contains an infinite set of fields of ever increasing spin which is, in spirit,

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reminiscent of the spectrum of string theory. This situation changed soon after the discovery of the supermembrane in 1987 [15]. A great deal of attention was given to the fact that its local fermionic symmetries require $D = 11$ supergravity equations of motion to be satisfied [15]. This led to the suggestion that supergravity in $D = 11$ could be considered as the low energy limit of a supermembrane theory, though, as we know, this issue is still not entirely well understood. In any event, the discovery of the supermembrane-supergravity connection motivated the study of the Kaluza-Klein vacua of $D = 11$ supergravity as vacua for the supermembrane, and special attention was given to the $AdS_4 \times S^7$ solution.

Soon after the discovery of the eleven-dimensional supermembrane, it was suggested in [16] that the singletons could play a role in its description. Subsequently, it was conjectured in [17] and [18] that a whole class of singleton/doubleton field theories constructed on the boundary of certain AdS_{p+1} spaces described super p-branes propagating on $AdS_{p+1} \times S^N$ which existed for certain p and N as supersymmetric Kaluza-Klein backgrounds of a class of supergravity theories. Yet, the $AdS_7 \times S^4$ compactification of $D = 11$ supergravity and the $AdS_5 \times S^5$ compactification of type-IIB supergravity in $D = 10$ were inexplicably overlooked; should they have also been considered, they would have pointed to the existence of the five-brane in $D = 11$ and the type-IIB three-brane in $D = 10$, several years before their actual discovery.

Once the idea of the eleven-dimensional supermembrane on $AdS_4 \times S^7$ being described by the singletons was entertained, it was natural to consider the possibility of AdS_4 higher spin fields arising in the spectrum of the supermembrane. In 1988, Bergshoeff, Salam, Sezgin and Tanii [19] proposed that the spectrum of the supermembrane in the $AdS_4 \times S^7$ background (treated as a second quantized singleton field theory in three dimensions) contains the massless higher spin states contained in the symmetric product of two $N = 8$ supersingletons. These states fill irreps of OSp(8|4) with highest spin $s_{\max} = 2, 4, 6, \ldots$, where the $s_{\max} = 2$ multiplet corresponds to the well-known gauged $D = 4, N = 8$ supergravity. It was also pointed out in [19] that the massive multiplets contained in the products of three or more singletons would appear in the spectrum. Moreover, it was realized in [19] that while the singleton field theory is free, it will nonetheless yield interactions in the bulk of AdS_4, in analogy with 2D free conformal field theory being capable of describing interactions in 10D target space. It was also suggested in [19] that the resulting theory in AdS_4 could provide a field theoretic realization of the infinite-dimensional higher spin algebras of the kind considered by Fradkin and Vasiliev [13]. Using the remarkable relation between the singletons and spectrum of higher spin states mentioned earlier, the “admissible” higher spin algebras were, in fact, determined later by Konstein and Vasiliev [20].

It is interesting that the massless higher representations would emerge first in the context of supermembrane in $AdS_4 \times S^7$ background, as opposed to string theory, perhaps arising from the Regge trajectory of massive states at high energies. In fact, in 1991, Fradkin and Linetsky [21], conjectured that “there might be some sort of phase transition in string theory at high energies when the cosmological constant of the Planck order is induced and an infinite-dimensional AdS higher spin gauge symmetry is restored". These authors were motivated by the similarity between the non-analyticity in string tension of the massive string states and the non-analyticity in cosmological constant of the higher spin gauge theory. In fact, an attempt was made in [21] to study string theory in AdS target space, with emphasis on determining the critical value of the cosmological constant to ensure freedom from worldsheet conformal anomalies. These authors also suggested in [21] the possibility of massless higher spin fields emerging from the product of two singleton states. In retrospect, one wonders why string theory in $AdS_5 \times S^5$ was not considered already then in this context.

Regardless of the considerations of a possible connection with strings or membranes, the theory of a consistent, interacting higher spin gauge theory initiated by Fradkin and Vasiliev [13, 14] was developed further by Vasiliev in a series of papers. In particular, spin zero and half fields were introduced to the system within the framework of free differential algebras [22]. The need to introduce these matter fields fitted nicely with the fact that they correspond precisely to the spin zero and half states that arise in the product of two singletons that carry the representation of the appropriate admissible higher spin algebra. The theory was furthermore cast into an elegant geometrical form in [23] by extending the higher spin algebra to include new auxiliary commuting spinorial variables. The resulting formulation of the theory is a free differential algebra containing a master gauge field and a master scalar field defined in an extended spacetime which has the usual four (commuting) spacetime coordinates as well as a set of non-commutative Grassmann even spinorial coordinates. The non-commutativity is defined by a star-product involving non-trivial contractions between the extended spinor coordinates as well as between these coordinates and the algebra oscillators. Solving the constraints in the extended directions and evaluating the remaining constraints at a subspace isomorphic to the ordinary spacetime leads to a deformed free differential algebra in the space time that includes interactions and that gives the correct free massless higher spin equations upon linearization. As such, the theory is realized at the level of field equations, but an action from which these field equations can be derived is not known.

The advances made in 1995 with the emergence of D-branes and M-theory, while highlighting the importance of branes and the role of eleven dimensions, did not revive interest in higher spin gauge theory. In fact, the eleven-dimensional membrane, which now was being referred to as the M2-brane, became one of the many possible branes that existed in a p-brane democracy, that had M5-branes, several D-branes, and other kinds of branes as well. As for the AdS background, although it was realized that certain brane solutions extrapolated between Minkowski spacetime and AdS space [24], the surprisingly powerful consequences of AdS background were really appreciated first in 1997 with Maldacena's conjecture [25] on the correspondence between physics in the bulk of AdS and conformal field theory on its boundary. As the

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main argument for the conjecture is that the AdS physics should actually be decoupled string theory or M-theory in the near horizon region of a brane (in some suitable limit), and as the precise formulation of string/M-theory in AdS spacetimes is still not under control, most attention has been focused, however, on the issue of testing a weaker form of the correspondence in the context of gauged supergravities, which are expected to describe the low energy limits of string/M-theory in AdS backgrounds.

This development motivated us to revisit the higher spin gauge theory [26, 27, 28]. The canonical examples of AdS/CFT correspondence are the maximally supersymmetric cases of type-IIB string theory in $AdS_5 \times S^5$ background and M-theory in $AdS_4 \times S^7$ and $AdS_7 \times S^4$ backgrounds. Since the higher spin gauge theory had been worked out in detail in AdS_4 and not yet in AdS_5 and AdS_7, we naturally studied further the case of AdS_4.

The main focus of our work in [26, 27] was to show how gauged $D = 4, N = 8$ supergravity is embedded in Vasiliev's higher spin gauge theory, and to elucidate the geometrical structure of these equations. While the embedding has been exhibited at the level of linearized field equations, interesting mysteries remain to be solved, as far as the non-linear embedding is concerned. Considering the simplest bosonic higher spin gauge theory, various aspects of a curvature expansion scheme advocated by Vasiliev [22] was re-visited in [28] but a detailed study of the non linearities in the theory is still lacking. In [26, 27], aspects of singleton dynamics on the boundary of AdS_4 yielding information on the higher spin gauge theory in the bulk were also discussed but were not put into a concrete mathematical foundation.

The case of AdS_5 appears to be more suitable, however, for examining the details of the AdS/CFT duality because the $N > 1$ version of the CFT is in principle known for this case, unlike the cases of AdS_4 and AdS_7, and large N is actually required for the AdS radius to be large compared to the Planck length, which is a basic requirement for the higher spin curvature expansion scheme to be reliable [28]. Of course, if one assumes that the 4D and yet to be constructed 7D higher spin gauge theories are actually contained in M-theory, one could infer the properties of large-N M2- and M5-brane dynamics from the corresponding higher spin curvature expansions. However, awaiting such a development, it is natural to focus our attention on the construction of a higher spin gauge theory in AdS_5.

Indeed, in an interesting recent development, the authors of [29, 30] have gathered evidence for that the physics of tensionless type-IIB strings in $AdS_5 \times S^5$ background involves massless higher spin fields. They also sketch a computational scheme for $N = 4$ supersymmetric Yang-Mills theory at zero coupling 't Hooft coupling and for large N, to support their arguments. This is to be contrasted with the original large-N and large 't Hooft coupling limit of Maldacena [25], in which case, as is well known, strongly coupled Yang-Mills theory furnishes a holographic description of type-IIB strings with finite tension. The latter setup has had only a limited scope, though, when it comes to actually verifying the AdS/CFT equivalence, due to the lack of computational schemes for the boundary and bulk theories. Indeed, mostly the implications of the gauged supergravity (valid at low energies) for the strongly coupled gauge theory on the boundary have been studied so far.

In the new limit proposed above we expect that computations can be performed on the higher spin supergravity side using the above-mentioned curvature expansion technique as well as on the boundary CFT side using the techniques of [29, 30]. Hence, this limit offers an arena for directly verifying the AdS/CFT conjecture! It remains, however, to find the interactions of the five-dimensional higher spin gauge theory, which we believe is an interesting and feasible technical problem. In this paper we have already taken the first steps in this directions by identifying an appropriate higher spin algebra in 5D and studying its gauging, thereby providing what we believe to be an appropriate framework for the construction of the full theory. Indeed, our results so far indicate that the theory has a form similar to the four-dimensional higher spin gauge theory. We shall return to the above issues in section 6.

While the higher spin gauge theory is well developed in $D = 4$, at least at the level of writing down full equations of motion in a concise and geometrical fashion, much less is known in $D > 4$. To a large extent this is due to the fact that the truly universal principles underlying the known four-dimensional case have not yet been identified completely. Once these principles are well understood, a natural strategy would, of course, be to apply them to any higher dimensions. This is the approach that we will take here. In fact, such a philosophy was also adopted by Vasiliev in 1990 [31] who considered some aspects of the problem in $D = 2n$, though the results do not appear to be conclusive. Much more is known, of course, about higher spin gauge theory in arbitrary dimensional AdS space at the free level [32]–[36]. The reason is that the linearized theory can be constructed without any knowledge of the underlying higher spin Lie algebra. One of the main aims of this paper is to remedy this situation in $D = 5$ (see below).

Another approach that has been proposed for studying the higher spin gauge theory problem in arbitrary dimensional AdS space is to consider a point particle in a higher spin gauge field background and to associate the higher spin gauge symmetry with the geometry of the point particle phase space [37]. So far, this approach seems to be rather restrictive and does not seem to make contact with Vasiliev's higher spin gauge theory in $D = 4$. However, recently an interesting connection has been made with the results of [37] for AdS_d, starting from a non-commutative $\mathrm{Sp}(2, R)$ gauge theory with two times in $(d, 2)$ dimensions and then fixing a particular gauge. It is suggested in [38] that some of the difficulties encountered in [37] may be overcome in their approach. Moreover, they also make a connection, as in [29, 30], between the higher spin gauge theory and the zero-tension limit of string theory.

The approach of [38] is certainly an interesting one to pursue. However, it is by no means clear at present how to reproduce even the existing higher spin gauge theory of Vasiliev in AdS_4 in that approach. Therefore, being armed with the knowledge we have gained from Vasiliev's theory in AdS_4, we choose in this paper the "building up approach" in which we try to carry over the basic principles of the construction that works in $D = 4$ to higher dimensions, beginning with AdS_5. In spirit, this is similar to search for a superspace formulation of supergravity field equations in terms of torsion and curvature constraints. One can then consider a superbrane action in curved superspace, whose κ symmetry would require that these constraints are satisfied. Historically, supergravity-brane connections have arisen in this manner. Here too, after establishing a higher spin gauge theory in terms of Vasiliev style constraints, one can search for a brane theory which will require those constraints. It would be rather amusing if one could discover the unknown higher spin gauge theory constraints by guessing an appropriate brane action to begin with and then requiring a suitable local symmetry.

The first principle in Vasiliev's approach to higher spin gauge theory in AdS background is to identify an appropriate higher spin gauge symmetry algebra. In doing so, we expect the fundamental representations of the AdS algebras which are known as singletons or doubletons to play a crucial role, just as they do in $D = 4$ [19, 20]. In this paper, we consider the bosonic higher in gauge theory in $D = 5$ and we find the suitable higher spin algebra, starting from the doubleton representations of the AdS_5 group $\mathrm{SO}(4,2)$ [39]. As in $D = 4$, it is obtained by using Grassmann even spinor variables, that are complex Dirac spinors of the spin extension $\mathrm{SU}(2,2)$ of $\mathrm{SO}(4,2)$, and we have therefore named it $\mathrm{hs}(2,2)$. The next step is to introduce suitable master fields which form representations of $\mathrm{hs}(2,2)$ and to define the associated curvatures and gauge transformations. Then, one has to find the suitable constraints on the curvatures such that their solution will give rise to certain auxiliary fields and dynamical fields, and moreover the latter ones should correspond to the spectrum of massless higher spin representations that arise in the tensor product of two doubletons. To ensure that the basic setup is right, we then check the resulting linearized field equations.[1]

We believe that in this paper we have established a framework for introducing interactions by deforming the linearized system. For a description of the deformation story, which is a crucial ingredient of the interacting theory as we understand it presently, see for example the reviews [27, 40]. We hope to return to the deformation problem in $D = 5$ in the future, by making use of the formalism established here. As for the generalization to the case of higher spin supergravity theory with 32 real supersymmetries in $D = 5$, we have carried out essentially the same steps as in this paper, and those results will appear elsewhere [42].

The organization of this paper is as follows: In section 2, we review the representation theory of $\mathrm{SO}(4,2)$, and in particular we discuss in great detail the doubleton

[1] At the algebraic level, the oscillator algebra offers competing options for defining the higher spin algebra and the master scalar fields, which all lead to the same field content. At the linearized level, the definitions made here reproduces the correct spectrum.

representations and derive the decomposition formula for the product that underlies the spectrum. In section 3, we define the bosonic higher spin algebra in $D=5$, which we call $\mathrm{hs}(2,2)$, and derive its massless unitary representation, which consists states with spins $s=0,2,4,\ldots$. In section 4, we gauge the algebra $\mathrm{hs}(2,2)$ by introducing a master gauge field, and show that its $\mathrm{SO}(4,2)$ field content agrees with the previous suggestions [32, 33] from linearized analysis. In addition we introduce a master scalar field which forms a representation of $\mathrm{hs}(2,2)$ and it is shown to be necessary to describe the physical scalar as well as the Weyl tensors, which are the on-shell non-vanishing components of the master curvature two-form. In section 5, we write the linearized constraints on the curvature two-form and the covariant derivative of the scalar master field. These are shown to be integrable and to yield correctly linearized field equations in AdS_5, namely the scalar Klein-Gordon equation and the curvature constraints on auxiliary and dynamical gauge fields written in the tensorial basis of [32, 33]. In section 6, we summarize our results and speculate over future directions; in particular on the prospects of a connection with string/M-theory. A more detailed discussion of possible connections between higher spin gauge theories and M-theory will be given elsewhere [42].

2. Elements of $\mathrm{SO}(4,2)$ representation theory

In this section we review some basic elements of the representation theory of $\mathrm{SO}(4,2)$ [39] that will be needed for the analysis of the higher spin algebra given in the next section. We also refine the work of [39] in the sense that we compute the multiplicity of the higher spin massless weight spaces that occur in the decomposition in the product of two spin zero doubletons. This data is necessary for the application to the five-dimensional higher spin gauge theory in order to deduce the precise form of the spectrum in the next section.

The five-dimensional AdS group and the four-dimensional conformal group are isomorphic to $\mathrm{SO}(4,2)$ ($A=0,\ldots,3,5,6$):

$$[M_{AB},M_{CD}]=-i(\eta_{BC}M_{AD}+3\text{ more})\,, \tag{2.1}$$

where $\eta_{AB}=\mathrm{diag}(-1,+1,+1,+1,+1,-1)$. The maximal compact subgroup of $\mathrm{SO}(4,2)$, or rather its spin extension $\mathrm{SU}(2,2)$, is $L^0=S(\mathrm{U}(2)\times\mathrm{U}(2))=\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{SU}(2)_{\mathrm{R}}\times\mathrm{U}(1)_E$, which can be taken to be represented by the generators ($i=1,2,3$):[2]

$$\begin{aligned}&\mathrm{SU}(2)_{\mathrm{L}}:\ L_i=\frac{1}{2}\left(M_{5i}+\frac{1}{2}\epsilon_{ijk}M_{jk}\right),\qquad \mathrm{SU}(2)_{\mathrm{R}}:\ R_i=\frac{1}{2}\left(-M_{5i}+\frac{1}{2}\epsilon_{ijk}M_{jk}\right),\\&\mathrm{U}(1)_E:\ E=M_{60}\,.\end{aligned} \tag{2.2}$$

[2]We define $P_a=M_{6a}$ ($a=0,\ldots,3,5$). The generator $P_0=E=M_{60}$ is the AdS energy in five dimensions. In four dimensions E is the conformal hamiltonian, while $D=M_{65}$ is the generator of dilatations.

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The remaining generators of $\mathrm{SO}(4,2)$ split into a space L^+ of energy-raising operators and a space L^- of energy-lowering operators, such that $[E, L^\pm] = \pm L^\pm$, $[L^+, L^+] = 0 = [L^-, L^-]$ and $[L^+, L^-] = L^0$. Thus unitary positive energy representations of $\mathrm{SO}(4,2)$ (with the reality condition $(M_{AB})^\dagger = M_{AB}$ implying $(L^+)^\dagger = L^-$) consist of weight spaces $D(j_{\mathrm{L}}, j_{\mathrm{R}}; E_0)$ formed by acting with L^+ on a space of ground states, or lowest weight states, $|j_{\mathrm{L}}, j_{\mathrm{R}}; E_0\rangle$ which are annihilated by L^- and form a representation of L^0 labeled by $(j_{\mathrm{L}}, j_{\mathrm{R}}; E_0)$. Such representations can be obtained by taking

$$M_{AB} = \frac{1}{2}\bar{y} \star \Sigma_{AB} y = \frac{1}{2}\bar{y}\Sigma_{AB} y\,, \tag{2.3}$$

where the four-component $\mathrm{SO}(4,1)$ Dirac spinor y_α and its conjugate $\bar{y}^\alpha$ obey the oscillator algebra[3]

$$\left[y_\alpha, \bar{y}^\beta\right]_* = 2\delta_\alpha^\beta\,, \qquad y_\alpha \star \bar{y}^\beta = y_\alpha \bar{y}^\beta + \delta_\alpha^\beta\,, \qquad \bar{y}^\alpha \star y_\beta = \bar{y}^\alpha y_\beta - \delta_\beta^\alpha\,, \tag{2.4}$$

where $\star$ denotes the operator product and $y_\alpha \bar{y}^\beta$ the Weyl ordered product, and

$$\Sigma_{ab} = -\frac{i}{2}\Gamma_{ab}\,, \qquad \Sigma_{a6} = -\frac{i}{2}\Gamma_a\,. \tag{2.5}$$

From $(\Sigma_{AB})^\dagger = -\Gamma^0 \Sigma_{AB} \Gamma^0$ it follows that $(M_{AB})^\dagger = M_{AB}$, and $E = \frac{1}{4} y^\dagger y$ shows that the energy is positive. The representation content of the oscillator Hilbert space can be listed by going to the standard representation of the Dirac matrices:

$$\Gamma^0 = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \Gamma^i = i\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \qquad \Gamma^5 = i\Gamma^0\Gamma^1\Gamma^2\Gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{2.6}$$

which splits y_α into the following pair of $\mathrm{SU}(2)$-covariant oscillators ($I = 1,2$, $P = 1,2$):

$$\begin{aligned} y_\alpha &= \sqrt{2}\begin{pmatrix} a^I \\ b_P \end{pmatrix}, \qquad \bar{y}^\alpha = \sqrt{2}\left(-a_I, b^P\right), \qquad a^I = (a_I)^\dagger\,, \qquad b^P = (b_P)^\dagger\,, \\ \left[a_I, a^J\right]_* &= \delta_I^J\,, \qquad \left[b_P, b^Q\right]_* = \delta_P^Q\,. \end{aligned} \tag{2.7}$$

The compact $\mathrm{SO}(4,2)$ generators are then given by:

$$\begin{aligned} L_i &= \frac{1}{2}\left(\sigma^i\right)_I{}^J L_J{}^I\,, \qquad R_i = \frac{1}{2}\left(\bar{\sigma}^i\right)^P{}_Q R^Q{}_P\,, \\ E &= \frac{1}{2}\left(a^I a_I + b^P b_P\right) = \frac{1}{2}(N_a + N_b + 2)\,, \end{aligned} \tag{2.8}$$

[3] The five-dimensional Dirac matrices Γ_a ($a = 0, \ldots, 5$) obey $\{\Gamma_a, \Gamma_b\} = 2\eta_{ab}$. We define the Dirac conjugate $\bar{y}^\alpha = (y^\dagger i\Gamma^0)^\alpha$ and the Majorana conjugate $\bar{y}_\alpha = \bar{y}^\beta C_{\beta\alpha}$. The anti-symmetric conjugation matrix $C_{\alpha\beta}$ obeys $C_{\alpha\beta}C^{\gamma\beta} = \delta_\alpha^\gamma$ and a reality condition such that $(\bar{\psi}\chi)^\dagger = \bar{\chi}\psi$, where we by definition set $(\psi\chi)^\dagger = \chi^\dagger\psi^\dagger$. The matrix $(\Gamma^a C)_{\alpha\beta}$ is anti-symmetric and $(\Gamma^{ab} C)_{\alpha\beta}$ is symmetric.

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where $\bar{\sigma}^i = -(\sigma^i)^\star$ and

$$\begin{aligned} L^I{}_J &= a^I \star a_J - \frac{1}{2}\delta^I_J N_a = a^I a_J - \frac{1}{2}\delta^J_I a^K a_K\,, \\ R^P{}_Q &= b^P \star b_Q - \frac{1}{2}\delta^P_Q N_b = b^P b_Q - \frac{1}{2}\delta^P_Q b^R b_R\,, \\ N_a &= a^I \star a_I = a^I a_I - 1\,, \qquad N_b = b^P \star b_P = b^P b_P - 1\,. \end{aligned} \tag{2.9}$$

The remaining SO$(4,2)$ generators are the energy-lowering operators L_{IP} and the raising operators L^{IP} given by:

$$L_{IP} = a_I \star b_P = a_I b_P\,, \qquad L^{IP} = a^I \star b^P = a^I b^P\,, \tag{2.10}$$

satisfying the algebra

$$\left[L_{IP}, L^{JQ}\right]_* = \delta^J_I R^Q{}_P + \delta^Q_P L^J{}_I + \delta^J_I \delta^Q_P E\,. \tag{2.11}$$

Upon letting $|0\rangle$ be the oscillator vacuum obeying

$$a_I|0\rangle = b_P|0\rangle = 0\,, \tag{2.12}$$

the lowest weight states of the oscillator Hilbert space are given by

$$|(j,0;j+1)\rangle = a^{I_1}\cdots a^{I_{2j}}|0\rangle\,, \qquad |(0,j;j+1)\rangle = b^{P_1}\cdots b^{P_{2j}}|0\rangle\,, \qquad j = 0, \frac{1}{2}, 1, \ldots\,. \tag{2.13}$$

The resulting weight spaces $D(j,0;j+1)$ and $D(0,j;j+1)$ are known as the SO$(4,2)$ doubleton representations, and correspond to the mode expansions (in a fixed gauge) of $D=4$ conformal tensors with SO$(3,1)$ spin j. These weight spaces are not sufficiently large, however, for constructing the mode expansions of AdS_5 tensors. The spectrum of such tensors is contained in the N-fold tensor products ($N>1$) of the doubleton representations. Such a tensor product is formally equivalent to the following oscillator algebra (r,s =1,. . . ,N):

$$\left[a_I(r), a^J(s)\right]_* = \delta^J_I \delta_{rs}\,, \qquad \left[b_P(r), b^P(s)\right]_* = \delta^Q_P \delta_{rs}\,. \tag{2.14}$$

The representation of SO$(4,2)$ on the tensor product space is then given by

$$M_{AB} = \sum_r M_{AB}(r)\,, \qquad M_{AB}(r) = \frac{1}{2}\bar{y}(r)\Sigma_{AB} y(r)\,. \tag{2.15}$$

It follows that $E = j_{\rm L} + j_{\rm R} + N$. For $N=2$, that is the two-fold tensor product, this yields massless representations of SO$(4,2)$. As a preparation for the next section, we compute the tensor product of two spin zero doubletons. The weight space $D(0,0;1)$ is spanned by the states

$$|0\rangle\,, \qquad a^I b^P|0\rangle\,, \qquad a^I a^J b^P b^Q|0\rangle\,, \qquad \ldots\,. \tag{2.16}$$

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To find the ground states we start from the following general expansion of a state $|\psi\rangle$ in the tensor product with fixed energy $E = n + 2$:

$$
\begin{aligned}
|\psi\rangle = \sum_{k=0}^{n} \psi^{(k)}_{I(k),P(k);J(n-k),Q(n-k)} a^{I_1}(1)\cdots a^{I_k}(1) b^{P_1}(1)\cdots b^{P_k}(1) \times \\
\times\, a^{J_1}(2)\cdots a^{J_{n-k}}(2) b^{Q_1}(2)\cdots b^{Q_{n-k}}(2)|0\rangle\ ,
\end{aligned} \tag{2.17}
$$

where we use a condensed notation such that $I(k) = I_1 \cdots I_k$ denotes k (symmetrized) indices. Acting on this state with the energy-lowering operators $L_{IP} = a_I(1)b_P(1) + a_I(2)b_P(2)$ should give zero, which amounts to the following set of equations:

$$
\begin{aligned}
n^2 \psi^{(n)}_{IJ(n-1),PQ(n-1)} + \psi^{(n-1)}_{IP;J(n-1),Q(n-1)} &= 0\,, \\
(n-1)^2 \psi^{(n-1)}_{KR;IJ(n-2),PQ(n-2)} + 4\psi^{(n-2)}_{IK,PR;J(n-2),Q(n-2)} &= 0\,, \\
&\ \vdots \\
\psi^{(1)}_{K(n-1),R(n-1);IP} + n^2 \psi^{(0)}_{IK(n-1),PR(n-1)} &= 0\,.
\end{aligned} \tag{2.18}
$$

From the first equation we can solve for $\psi^{(n-1)}$ in terms of $\psi^{(n)}$, etc. The ground states with $E = n + 2$ form an irreducible representation of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ with quantum numbers $(j_L, j_R) = (n/2, n/2)$. The lowest energy states listed in the order of increasing energy are

$$
\begin{aligned}
|(0,0;2)\rangle &= |0\rangle\,, \\
\left|\left(\frac{1}{2},\frac{1}{2};3\right)\right\rangle &= \big(a^I(1)b^P(1) - a^I(2)b^P(2)\big)\,|0\rangle\,, \\
|(1,1;4)\rangle &= \big(a^I(1)a^J(1)b^P(1)b^Q(1) - 4a^{(I}(1)a^{J)}(2)b^{(P}(1)b^{Q)}(2) + \\
&\quad + a^I(2)a^J(2)b^P(2)b^Q(2)\big)\,|0\rangle\,, \\
&\ \vdots \\
|(j,j;2j+2)\rangle &= \sum_{k=0}^{2j}(-1)^k \binom{2j}{k}^2 a^{(I_1}(1+\theta(k))\cdots a^{I_{2j})}(1+\theta(k+1-2j)) \times \\
&\quad \times b^{(P_1}(1+\theta(k))\cdots b^{P_{2j})}(1+\theta(k+1-2j))\ |0\rangle\,, \\
&\ \vdots
\end{aligned} \tag{2.19}
$$

where $\theta(x) = 0$ if $x \le 0$ and $\theta(x) = 1$ if $x > 0$. The states with even spins $s = j_L + j_R = 0, 2, 4, \ldots$ belong to the symmetric tensor product and the states with odd spins $s = 1, 3, 5, \ldots$ to the anti-symmetric product:

$$
\begin{aligned}
[D(0,0;1) \otimes D(0,0;1)]_{\mathrm{S}} &= \sum_{s=0,2,\ldots} D\left(\frac{s}{2},\frac{s}{2};s+2\right)\,, \\
[D(0,0;1) \otimes D(0,0;1)]_{\mathrm{A}} &= \sum_{s=1,3,\ldots} D\left(\frac{s}{2},\frac{s}{2};s+2\right)\,.
\end{aligned} \tag{2.20}
$$

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3. The higher spin algebra $\mathrm{hs}(2,2)$ and its spectrum

In this section we define a higher spin extension $\mathrm{hs}(2,2)$ of $SO(4,2)$ by the coset $\mathcal{G}/\mathcal{I}$, where $\mathcal{G}$ is a Lie subalgebra of the algebra $\mathcal{A}$ of arbitrary polynomials of the oscillators (2.4) and $\mathcal{I}$ is an ideal of $\mathcal{G}$ generated by a central element K. The basic argument for modding out K is that it is responsible for a degeneracy in $\mathcal{G}$ such that $\mathcal{G}$ contains infinitely many generators of any given integer spin. The reason for this is that K has zero spin so that it can be used to build elements in $\mathcal{G}$ of arbitrary monomial degree but with fixed spin. On the other hand, the coset defining $\mathrm{hs}(2,2)$ has a finite number of generators of any given spin. In this section, we also define the physical spectrum $\mathcal{S}$ of the five-dimensional higher spin gauge theory based on $\mathrm{hs}(2,2)$. The basic requirement on $\mathcal{S}$ is that it must consist of massless $SO(4,2)$ weight spaces and carry a unitary (irreducible) representation of $\mathrm{hs}(2,2)$. Armed with the algebra $\mathrm{hs}(2,2)$ and its massless spectrum $\mathcal{S}$, we will gauge $\mathrm{hs}(2,2)$ in the next section.

To define the algebra we first define the associative product of elements in $\mathcal{A}$, that is Weyl-ordered (regular) functions of the oscillators y and $\bar{y}$, as follows:

$$F(y,\bar{y})\star G(y,\bar{y})=\int d^8u d^8v F(y+u,\bar{y}+\bar{u})G(y+v,\bar{y}+\bar{v})e^{\bar{u}v-\bar{v}u}\,, \tag{3.1}$$

where the integration measure is assumed to be normalized such that $1\star F=F\star 1=F$. This algebra can also be defined by the following contraction rule:

$$\begin{aligned}
&(y_{\alpha_1}\cdots y_{\alpha_m}\bar{y}^{\beta_1}\cdots\bar{y}^{\beta_n})\star(y_{\gamma_1}\cdots y_{\gamma_p}\bar{y}^{\delta_1}\cdots\bar{y}^{\delta_q})=\\
&\quad= y_{\alpha_1}\cdots y_{\alpha_m}\bar{y}^{\beta_1}\cdots\bar{y}^{\beta_n}y_{\gamma_1}\cdots y_{\gamma_p}\bar{y}^{\delta_1}\cdots\bar{y}^{\delta_q}+\\
&\qquad+\ mq\delta^{(\delta_1|}_{(\alpha_1}y_{\alpha_2}\cdots y_{\alpha_m)}\bar{y}^{\beta_1}\cdots\bar{y}^{\beta_n}y_{\gamma_1}\cdots y_{\gamma_p}\bar{y}^{|\delta_2}\cdots\bar{y}^{\delta_q)}-\\
&\qquad-\ np\delta^{(\beta_1}_{(\gamma_1|}y_{\alpha_1}\cdots y_{\alpha_m}\bar{y}^{\beta_2}\cdots\bar{y}^{\beta_n)}y_{|\gamma_2}\cdots y_{\gamma_p)}\bar{y}^{\delta_2}\cdots\bar{y}^{\delta_q}+\\
&\qquad+\ \frac{m(m-1)q(q-1)}{2}\delta^{(\delta_1\delta_2|}_{(\alpha_1\alpha_2}y_{\alpha_3}\cdots y_{\alpha_m)}\bar{y}^{\beta_1}\cdots\bar{y}^{\beta_n}y_{\gamma_1}\cdots y_{\gamma_p}\bar{y}^{|\delta_3}\cdots\bar{y}^{\delta_q)}+\\
&\qquad+\ \cdots\,.
\end{aligned} \tag{3.2}$$

A general term obtained by contracting k $y\bar{y}$ pairs and l $\bar{y}y$ pairs is weighted with

$$(-1)^l k!l!\binom{m}{k}\binom{q}{k}\binom{n}{l}\binom{p}{l}\delta^{\delta_1\cdots\delta_k}_{\alpha_1\cdots\alpha_k}\delta^{\beta_1\cdots\beta_l}_{\gamma_1\cdots\gamma_l}\,. \tag{3.3}$$

Here we use unit-strength symmetrized Kronecker-deltas defined by $\delta^{\alpha_1\cdots\alpha_p}_{\beta_1\cdots\beta_p}=\delta^{(\alpha_1}_{(\beta_1}\cdots\delta^{\alpha_p)}_{\beta_p)}$. Since $(\bar{u}v)^\dagger=\bar{v}u$ it follows from (3.1) that:

$$(F\star G)^\dagger=G^\dagger\star F^\dagger\,. \tag{3.4}$$

The following set of linear maps:

$$\tau_\eta(y_\alpha)=\eta y_\alpha\,,\qquad \tau_\eta(\bar{y}_\alpha)=-\bar{\eta}\bar{y}_\alpha\,,\qquad |\eta|=1\,, \tag{3.5}$$

act as anti-involutions of $\mathcal{A}$:

$$\tau_\eta(F \star G) = \tau_\eta(G) \star \tau_\eta(F) \,. \tag{3.6}$$

The Lie subalgebra $\mathcal{G}$ is defined to be the subspace of $\mathcal{A}$ consisting of elements F obeying ($|\eta| = 1$)

$$\tau_\eta(F) = -F \,, \qquad (F)^\dagger = -F \,, \tag{3.7}$$

and with Lie bracket

$$[F, G] = [F, G]_* = F \star G - G \star F \,. \tag{3.8}$$

Lie algebras that are similar to $\mathcal{G}$ have been defined in even spacetime dimensions by Vasiliev in a slightly different setup [31]. The algebra $\mathcal{G}$ can be expanded in terms of elements of the form:

$$\frac{1}{(n!)^2} X_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n} \bar{y}^{\alpha_1}\cdots\bar{y}^{\alpha_n} y^{\beta_1}\cdots y^{\beta_n} \,, \qquad n = 1, 3, 5, \ldots \,, \tag{3.9}$$

where the multi-spinor coefficient obeys the following reality condition:

$$\bar{X}_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n} \equiv \bar{X}^{\gamma_1\cdots\gamma_n,\delta_1\cdots\delta_n} C_{\gamma_1\alpha_1}\cdots C_{\delta_n\beta_n} = -X_{\beta_1\cdots\beta_n,\alpha_1\cdots\alpha_n} \,. \tag{3.10}$$

Note that the Dirac conjugate multi-spinor $\bar{X}^{\gamma_1\cdots\gamma_n,\delta_1\cdots\delta_n}$ is defined by a hermitean conjugation followed by multiplication with $i\Gamma^0$ of each spinor index. The elements in (3.9) with $n = 1$ form the subalgebra $\mathrm{U}(2,2) = \mathrm{SU}(2,2) \times \mathrm{U}(1)_K$, where $\mathrm{U}(1)_K$ is generated by the central element

$$K = \frac{1}{2}\bar{y}y \,, \qquad [K, F]_* = 0 \,, \qquad F \in \mathcal{G} \,. \tag{3.11}$$

From (2.7) and (2.9) it follows that

$$K = \frac{1}{2}\left(-a^I a_I + b^P b_P\right) = \frac{1}{2}(N_b - N_a) \,. \tag{3.12}$$

In a unitary irreducible representation of $\mathcal{G}$ the generator K is given by a real constant. In particular, the algebra $\mathcal{G}$ can be represented unitarily on the oscillator Fock space. As discussed in the previous section, the oscillator Fock space decomposes into a direct sum of all the doubletons $D(j, 0; j+1)$ and $D(0, j; j+1)$ ($j = 0, 1/2, 1, 3/2, \ldots$). By construction each doubleton is an irreducible representations of the $\mathrm{U}(2,2)$ subalgebra, in which

$$K = \begin{cases} \frac{j}{2} \,, & \text{for} \quad D(0, j; j+1) \,, \\ -\frac{j}{2} \,, & \text{for} \quad D(j, 0; j+1) \,. \end{cases} \tag{3.13}$$

It follows that each doubleton is also a unitary irreducible representation of $\mathcal{G}$.

To examine the degeneracy due to the fact that K has spin zero, we decompose $\mathcal{G}$ into levels, such that the ℓth level is given by all elements of the form:

$$\begin{aligned}
&\frac{1}{(n!)^2} K^{\star k} \star \left(X^{(k)}_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n} \bar{y}^{\alpha_1} \cdots \bar{y}^{\alpha_n} y^{\beta_1} \cdots y^{\beta_n} \right) , \\
&C^{\gamma\delta} X^{(k)}_{\gamma\alpha_1 \cdots \alpha_{n-1}, \delta\beta_1 \cdots \beta_{n-1}} = 0 \, , \qquad k+n = 2\ell+1 \, , \qquad k, n \geq 0 \, ,
\end{aligned} \tag{3.14}$$

where we emphasize that $X^{(k)}$ is traceless (recall that $C_{\alpha\beta}$ is the anti-symmetric charge conjugation matrix in $D=5$), and we use the notation

$$K^{\star k} = \underbrace{K \star \cdots \star K}_{k \text{ factors}} . \tag{3.15}$$

For finite polynomials the expansions (3.9) and (3.14) are equivalent. By making repeated use of (3.1) and (3.14) can be expanded as a leading term $\frac{1}{(n!)^2} K^k X^{(k)}_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n}$ $\bar{y}^{\alpha_1} \cdots \bar{y}^{\alpha_n} y^{\beta_1} \cdots y^{\beta_n}$ plus a finite number of terms of lower polynomial degree; that is, the basis (3.14) corresponds to separating out the $C_{\alpha\beta}$ traces of the basis elements in (3.9). Note that an element in the ℓth level is a sum of elements of the form (3.9) with $n \leq 2\ell+1$. The basis (3.14) yields the following unique decomposition of $\mathcal{G}$:

$$\mathcal{G} = \mathcal{G}^{(0)} + K \star \mathcal{G}^{(1)} + K^{\star 2} \star \mathcal{G}^{(2)} + \cdots . \tag{3.16}$$

Since K is central and $\tau_\eta(K) = -K$, it follows from (3.7) that $\tau(X^{(k)}) = (-1)^{1+k} X^{(k)}$. Hence $\mathcal{G}^{(k)}$ is isomorphic to $\mathcal{G}^{(0)}$ or $\mathcal{G}^{(1)}$ for k even or odd, respectively. We also remark that since K is central and $K^\dagger = K$, the traceless multi-spinors $X^{(k)}$ obey the reality condition (3.10).

The degeneracy in $\mathcal{G}$ discussed above due to K having spin zero suggests that K should be eliminated from the actual higher spin algebra. The Lie bracket (3.8) induces a set of brackets with the following structure:

$$[\cdot,\cdot] \;:\; \mathcal{G}^{(k_1)} \times \mathcal{G}^{(k_2)} \;\longrightarrow\; \mathcal{G}^{(k_1+k_2)} + \mathcal{G}^{(k_1+k_2+1)} + \cdots . \tag{3.17}$$

Here the direct sum, which is finite, is due to the fact that the Lie bracket (3.8) does not preserve the tracelessness condition in (3.14). Thus K cannot be eliminated by simply restricting $\mathcal{G}$ to $\mathcal{G}^{(0)}$. In order to factor out K we instead let

$$\mathcal{I} = K \star \mathcal{G}^{(1)} + K^{\star 2} \star \mathcal{G}^{(2)} + \cdots . \tag{3.18}$$

This space forms an ideal in $\mathcal{G}$, i.e. $[\mathcal{G}, \mathcal{I}]_* = \mathcal{I}$. We can now define the higher spin algebra $\mathrm{hs}(2,2)$ as following coset:

$$\mathrm{hs}(2,2) = \frac{\mathcal{G}}{\mathcal{G}'} . \tag{3.19}$$

The elements of $\mathrm{hs}(2,2)$ are thus equivalence classes $[F]$ of elements in $\mathcal{G}$ defined by

$$[F] = \{ G \in \mathcal{G} \mid F - G \in \mathcal{I} \} . \tag{3.20}$$

The Lie bracket of $[F]$ and $[G]$ is given by

$$[[F],[G]] = [[F,G]_*] \,. \tag{3.21}$$

The spectrum $\mathcal{S}$ of $D=5$ higher spin gauge theory based on hs$(2,2)$ should be a unitary representation of hs$(2,2)$ that decomposes into massless weight spaces under SO$(4,2)$. This condition is necessary, provided that the theory has an expansion around a maximally symmetric AdS vacuum (since the vacuum must in fact be hs$(2,2)$ invariant). Moreover, in order for hs$(2,2)$ to have a well-defined action on $\mathcal{S}$ we must demand that $K=0$ in $\mathcal{S}$. This shows that $\mathcal{S}$ must be made up of tensor products of two spin j doubletons with opposite eigenvalue of K, that is $[D(j,0;j+1)\otimes D(0,j;j+1)]_{\rm S,A}$.

In order to determine which values of j contribute to $\mathcal{S}$, we can study the gauging of hs$(2,2)$ and examine the resulting curvature constraints (i.e. generalizations of the spin two Einstein equation) at the linearized level, which would yield information of the spin $s\geq 2$ sector of $\mathcal{S}$. Incorporating the spin $s\leq 1$ sector in an hs$(2,2)$ symmetric fashion amounts to introducing a scalar master field in some representation R of hs$(2,2)$. The uncertainty in the choice of R implies, however, that there is an uncertainty also in the spin $s\geq 2$ sector, since it is possible that R contains not just physical spin $s\leq 1$ fields but also physical spin $s\geq 2$ fields. This is in fact the case in supersymmetric extensions of this bosonic model, as we shall comment on in section 6. Thus, a determination of $\mathcal{S}$ based on gauging alone may have to involve a rather elaborate ansatz, unless one is willing to accept some loss of generality or one invokes some other basic principle.

In order to determine $\mathcal{S}$ we assume that the hs$(2,2)$ gauge theory is some limit of string theory. Since the gauge theory has an AdS vacuum, we assume that it describes a bosonic truncation of the residual type-IIB string bulk dynamics in the near-horizon region of N coincident three-branes in a decoupling limit in which the $4D$ conformal symmetry group is enhanced to hs$(2,2)$. The precise definition of this limit, which was suggested recently by [29, 30], is discussed further in section 6. We are thus led to imposing the additional requirement that the spectrum-generating doubleton representations form a unitary irreducible representation of hs$(2,2)$. From the above analysis it follows that this uniquely selects the spin $j=0$ representation $D(0,0;2)$. In the above limit, the bosonic truncation of the boundary theory is a 4D free scalar in the adjoint representation of the (global) SU(N) symmetry. The massless higher spin states emerge in the sector of bilinears in the scalar field and its derivatives which can be written as single traces [29, 30]. For example, the mass-operator gives rise to a scalar state, while the remaining states corresponds to a set of higher spin currents [40, 41]. Importantly, the spectrum has no spin one state, as the corresponding current is a descendant (total derivative) of the mass-operator. This implies that the massless higher spin spectrum $\mathcal{S}$ is given by the symmetric tensor product:

$$\mathcal{S} = [D(0,0;1) \otimes D(0,0;1)]_{\mathrm{S}} \; . \tag{3.22}$$

It follows from (2.20) that $\mathcal{S}$ consists of the physical states of five-dimensional massless fields with spins $s = 0, 2, \dots$ and energies $E = s+2$. The anti-symmetric part of the tensor product contains states with odd spins, which from the boundary point of view correspond to states which are descendants, as explained above in the case of spin one. We remark that from the point of view of reconstructing the bulk theory from a boundary theory which has vanishing K, it is more natural to mod out the central U(1) generator K from the bulk theory than setting it equal to zero.[4]

4. Gauging hs(2, 2)

In order to realize hs(2, 2) as a local symmetry in a field theory with spectrum $\mathcal{S}$ we need to address the following two basic issues. Firstly, gauging of hs(2, 2) introduces both dynamic gauge fields and auxiliary gauge fields. Fortunately the structure of a set of gauge fields and curvature constraints that give rise to one massless spin s degree of freedom are known at the linearized level in an expansion around AdS spacetime, albeit in SO(4, 1) basis, instead of the spinor basis introduced in the previous section. Thus in order to give the linearized hs(2, 2) valued constraints it suffices to find a one-to-one map between these two bases.

Secondly, the spectrum $\mathcal{S}$ in (3.22) contains a spin zero state.[5] In order to incorporate this degree of freedom while retaining manifest hs(2, 2) gauge invariance, it is natural to generalize Vasiliev's four-dimensional formulation of higher spin theory and identify the spin zero mode with the leading component of a scalar master field Φ in a particular representation of hs(2, 2) to be identified below. Its remaining components should be the components of the curvature that are non-vanishing on-shell, that in the spin two case is known as the Weyl tensor and that are referred to as the generalized Weyl tensors in the cases of higher spin, as well as the derivatives of the scalar field and the generalized Weyl tensors.

In the remainder of this section we are concerned with establishing the equivalence between the spinorial basis (3.14) for hs(2, 2) and the tensorial basis of [32, 33] and to give the definition of the scalar master field. The linearized analysis is given in the next section.

[4] It is also possible to eliminate K from $\mathcal{G}$ by going to the Lie algebra of elements obeying $K \star F = 0$, $F \in \mathcal{G}$. This equation can be solved by an infinite expansion in K^2 using elements of the form (3.9); see (4.19) and below. This Lie algebra gives rise to the same field content as hs(2, 2) upon gauging, but it seems unnatural from the boundary point of view and leads to undesirable complications of the algebra as well. We expect that it can be ruled out at the linearized level; see also footnote 6.

[5] In the $N = 4$ supersymmetric case this state becomes the lowest spin state of a spin four multiplet, as explained in section 6.

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We begin by introducing the $\mathcal{G}$ valued one-form ($|\eta|=1$)

$$A = dx^\mu A_\mu(y,\bar{y})\,, \qquad \tau_\eta(A) = -A\,, \qquad (A)^\dagger = -A\,, \tag{4.1}$$

and a zero-form B in the following representation $\mathcal{R}$ of $\mathcal{G}$ ($|\eta|=1$):

$$\tau_\eta(B) = \pi(B)\,, \qquad (B)^\dagger = \pi(B)\,. \tag{4.2}$$

Here π is the linear map

$$\pi(y_\alpha) = \bar{y}_\alpha\,, \qquad \pi(\bar{y}_\alpha) = y_\alpha\,, \tag{4.3}$$

which acts as an involution of the algebra $\mathcal{A}$:

$$\pi(F\star G) = \pi(F)\star\pi(G)\,. \tag{4.4}$$

The $\mathcal{G}$ gauge transformations are given by:

$$\delta_\epsilon A = d\epsilon + [A,\epsilon]_*\,, \qquad \delta_\epsilon B = B\star\pi(\epsilon) - \epsilon\star B\,, \tag{4.5}$$

where ϵ is a $\mathcal{G}$ valued local parameter, such that the following curvature and covariant derivative obey (4.1) and (4.2) and are $\mathcal{G}$ covariant:

$$F_A = dA + A\wedge\star A\,, \qquad \delta_\epsilon F_A = [F_A,\epsilon]_\star\,, \tag{4.6}$$

$$D_A B = dB - B\star\pi(A) + A\star B\,, \qquad \delta_\epsilon D_A B = D_A B\star\pi(\epsilon) - \epsilon\star D_A B\,. \tag{4.7}$$

To show that $\delta_\epsilon B$ and $D_A B$ obey (4.2) one needs to use (3.4), (3.6) and (4.4), $\pi^2=1$ and that $\tau_\eta(\pi(F)) = -\pi(F)$, $(\pi(F))^\dagger = -\pi(F)$ for $F\in\mathcal{G}$. For example, to show that $\delta_\epsilon(B)$ obey $\tau_\eta(\delta_\epsilon(B)) = \pi(\delta_\epsilon(B))$ we compute:

$$\begin{aligned}\tau_\eta(\delta_\epsilon B) &= \tau_\eta(B\star\pi(\epsilon) - \epsilon\star B) = \tau_\eta(\pi(\epsilon))\star\tau_\eta(B) - \tau_\eta(B)\star\tau_\eta(\epsilon)\\ &= -\pi(\epsilon)\star\pi(B) + \pi(B)\star\epsilon = \pi(B\star\pi(\epsilon) - \epsilon\star B) = \pi(\delta_\epsilon B)\,.\end{aligned} \tag{4.8}$$

We also remark that a similar calculation shows that B indeed belongs to a representation of $\mathcal{G}$, that is

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]B = \delta_{[\epsilon_1,\epsilon_2]_*}B\,. \tag{4.9}$$

Next we define the hs$(2,2)$-valued gauge field, curvature and gauge parameter by

$$[A]\,, \qquad F_{[A]} = [F_A]\,, \qquad [\epsilon]\,, \tag{4.10}$$

where we use the notation of (3.20). The gauge transformations read:

$$\delta_{[\epsilon]}[A] = [\delta_\epsilon A]\,, \qquad \delta_{[\epsilon]}F_{[A]} = [\delta_\epsilon F_A]\,. \tag{4.11}$$

By construction the above expressions are independent of the choice of representative in $\mathcal{G}$ of the various hs$(2,2)$-valued quantities. We note that the curvature and the

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gauge transformations are computed by first evaluating the ordinary $\star$ product (3.1) between the representatives and then expanding the result with respect to the particular ordering of oscillators defined by (3.14) and finally discarding any terms in $\mathcal{I}$. In case one would have to perform several repeated multiplications of objects in $hs(2,2)$ the last step may of course be carried out at the end, as the operation of modding out K commutes with taking the $\star$ product.

The $hs(2,2)$-valued gauge field $[A]$ can be represented by a $\mathcal{G}^{(0)}$-valued gauge field A, which has an expansion in terms of component fields with tangent indices corresponding to the traceless multispinors $X^{(0)}$ defined in (3.14), obeying the reality condition (3.10). The level ℓ generators of $\mathcal{G}^{(0)}$ thus gives rise to component gauge field with spin $s = 2\ell + 2$, where s is defined to be 1 plus the internal spin, which equals $2\ell + 1$ in the ℓth level.

A real and $C_{\alpha\beta}$ traceless multi-spinor $T_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n}$, ($n = 2\ell + 1 = s - 1$ at the ℓth level of $\mathcal{G}^{(0)}$), can be decomposed into irreducible multi-spinors $T^{(p;0)}_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n}$ with index structures corresponding to the SU(2,2) Young tableaux:

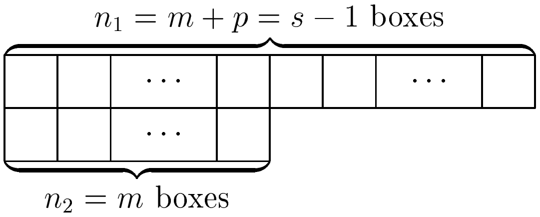
$$T^{(p;0)}_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n} = \underbrace{\begin{array}{|c|c|c|}\hline \bullet & \cdots & \bullet \\ \hline & \cdots & \\ \hline\end{array}}_{p\text{ boxes}}\overbrace{\begin{array}{|c|c|c|c|c|}\hline \bullet & & \cdots & \bullet & \\ \hline\end{array}}^{2m\text{ boxes}}\;, \qquad p+m=n\,, \tag{4.12}$$

where the "undotted" and "dotted" boxes refer to spinor indices contracted with y spinors and $\bar{y}$ spinors, respectively. These spinors belong to equivalent representations of SU(2,2), and hence their indices can be put in the same Young tableaux. Since the spinors are Grassmann even two dotted or undotted boxes cannot be placed on top of each other. To count the (real) dimension $d_{p,m}$ of the Young tableaux (4.12) we thus first compute the complex dimension $D_{p,m}$ by performing a "SU(4)-count" in which the dotted and undottedness is neglected. These correspond to imposing the reality condition (it is not important whether T is real or purely imaginary), which implies $d_{p,m} = D_{p,m}$. We remark that the reality condition of course requires SU(2,2) spinors. Taking also the tracelessness condition in (3.14) into account one finds that the dimension of (4.12) is given by

$$\begin{aligned} d_{p,m} &= \frac{4\cdot 5\cdots(3+2m+p)\cdot 3\cdot 4\cdots(2+p)}{(2m+p+1)\cdot(2m+p)\cdots(2m+1)\cdot(2m)\cdot(2m-1)\cdots 1\cdot p\cdot(p-1)\cdots 1} \\ &\quad - \text{same with } p\to p-1 \\ &= \frac{1}{12}(p+2)(p+1)(2m+1)(2m+p+2)(2m+p+3) - \text{same with } p\to p-1 \\ &= \frac{2}{3}\left(m+p+\frac{3}{2}\right)\left(m+\frac{1}{2}\right)(2m+p+2)(p+1)\,. \end{aligned} \tag{4.13}$$

It has been shown [32, 33] that linearized curvature constraints (see section 5) leading to the on-shell massless spin s weight space $D(s/2, s/2; s+2)$ of $\mathrm{SO}(4,2)$ can be written using a space of five-dimensional gauge fields with tangent space indices given by irreducible $\mathrm{SO}(4,1)$ tensors $T^{(p,m)}_{a_1b_1,\dots,a_mb_m;c_1\cdots c_p}$, $m+p=s-1$, corresponding to the following Young tableaux:

$$T^{(p,m)}_{a_1b_1,\dots,a_mb_m;c_1\cdots c_p} =$$



$$, \qquad 0 \le n_1 \le n_2\,, \tag{4.14}$$

Here the notation is such that the pair of indices a_ib_i $(i=1,\dots,m)$ goes into the ith pair of anti-symmetrized boxes and $c_1\cdots c_p$ into the remaining p symmetrized boxes. The irreducible Young tableaux (4.14) has dimension

$$d'_{p,m} = \frac{2}{3}\left(n_1+\frac{3}{2}\right)\left(n_2+\frac{1}{2}\right)(n_1+n_2+2)(n_1-n_2+1)\,. \tag{4.15}$$

The dimensions (4.13) and (4.15) agree for $n=s-1$, so that (4.14) can be converted into (4.12) by making use of Dirac matrices. Thus the spin s gauge fields in the spinorial basis are in one-to-one correspondence with the spin s gauge fields in the lorentzian basis of [32, 33]. We emphasize, however, that whereas the latter formulation is only defined in a linearization around AdS, and thus does not contain any information about the full higher spin gauge algebra, this data are naturally incorporated in the spinorial formalism used here.

The master field B can be expanded in terms of elements of the form

$$B^{(r;t)}(n) = \frac{1}{(n!)^2} B^{(r;t)}_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n}\bar{y}^{\alpha_1}\cdots\bar{y}^{\alpha_n}y^{\beta_1}\cdots y^{\beta_n} = K^t B^{(r-t;0)}_t(n-t)\,, \tag{4.16}$$

$$r,t,n \ge 0\,, \qquad n-r = 0,2,4,\dots\,, \tag{4.17}$$

where the superscript r denotes the number of anti-symmetric pairs of spinor indices and t $(0 \le t \le r)$ the number of these that are traced. From (4.2) it follows that the multi-spinors in (4.16) obey the following reality condition:

$$\bar{B}^{(r;t)}_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n} = B^{(r;t)}_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n}\,. \tag{4.18}$$

Modulo the degeneracy due to K, the field content of the master scalar field therefore falls into "trajectories" $B^{(p;0)}(m+p)$, $p=0,1,2,\dots$, for each $m=0,2,\dots$. The one-to-one map between (4.12) and (4.14) shows that the leading component $B^{(0;0)}(m)$ defines a traceless spin $s=m$ tensor carrying s pairs of anti-symmetric $\mathrm{SO}(4,1)$ indices. For $s=2,4,\dots$ this is exactly the index structure of the spin s Weyl tensor that was introduced in the linearized analysis of [32, 33], while $B^{(0;0)}(0)$ is the desired (real) scalar field. For $p>0$ the index structure $B^{(p;0)}(m+p)$ is exactly that of p derivatives of the leading component of the trajectory. Thus, apart from the degeneracy, the desired field content of the master scalar field emerges in B.

Next, we proceed by defining the master scalar field Φ in the representation R of $\mathrm{hs}(2,2)$, where R is the subspace of the representation space $\mathcal{R}$ of $\mathcal{G}$ defined by[6]

$$K\star\Phi=0\,,\qquad \Phi\in\mathcal{R}\,. \tag{4.19}$$

This condition serves two purposes. Firstly, it assures that the $\mathrm{hs}(2,2)$-covariant derivative and $\mathrm{hs}(2,2)$-gauge transformations of Φ, which are given by

$$D_{[A]}\Phi=d\Phi+\Phi\star\pi(A)-A\star\Phi\,,\qquad \delta_{[\epsilon]}\Phi=\Phi\star\pi(\epsilon)-\epsilon\star\Phi\,, \tag{4.20}$$

are well defined, i.e. independent of the choice of representative for $[A]$ and $[\epsilon]$ and obeying $K\star D_{[A]}\Phi=K\star\delta_{[\epsilon]}\Phi=0$. To see this we use (4.4) and $\pi(K)=-K$ and the fact that K is central. Secondly, (4.19) removes the degeneracy due to K. To see this we first solve (4.19) by using the following lemma:

$$K*\left(K^kT^{(r;0)}(n)\right)=\left(K^{k+1}-\frac{1}{4}k(k+2n+3)K^{k-1}\right)T^{(r;0)}(n)\,, \tag{4.21}$$

where $T^{(r;0)}(n)$ is a traceless multi-spinor given in the notation of (4.17). We note that in computing (4.21) the single-contractions cancel while the double-contractions are of three types: those involving K^k, which give a factor of $-\frac{1}{4}k(k+3)$; the mixed ones, which give $2(-\frac{1}{4}kn)$; and those involving T, which are proportional to the (vanishing) trace. Using (4.21) the condition (4.19) can solved recursively, leading to the following general solution:

$$\begin{aligned}\Phi^{(p;0)}_{2t}(n)&=\frac{1}{t!\left[t+n+\frac{3}{2}\right]_k}\Phi^{(p;0)}_0(n)\,,\\ \Phi^{(p;0)}_{2t+1}(n)&=0\,,\end{aligned} \tag{4.22}$$

where we use the notation of (4.16) and the Pochhammer symbol $[a]_b\equiv a(a-1)\cdots(a-b+1)$, for b positive integer. Thus the degeneracy is completely removed.

[6] It is also possible to eliminate K from the master scalar field by considering an expansion of B in terms of elements of the form $(K*)^t*B^{(p;0)}(n)$ instead of (4.16). This is analogous to the definition of the basis (3.14) for $\mathcal{G}$. This leads to a unique decomposition of $\mathcal{R}=\mathcal{R}^{(0)}+\mathcal{R}'$, where $\mathcal{R}'=\mathcal{R}^{(1)}+\mathcal{R}^{(2)}+\cdots$. The space $\mathcal{R}'$ is an $\mathcal{G}$ invariant subspace. Thus $\mathcal{R}/\mathcal{R}'$ is a representation space for $\mathrm{hs}(2,2)$ and we can define the $\mathrm{hs}(2,2)$ transformations and the covariant derivative by $\delta_{[\epsilon]}[B]=[\delta_\epsilon B]$ and $D_{[A]}[B]=[D_AB]$. However, using these results, a careful analysis of the scalar field equation shows that while (5.21) still holds, the trace term on the right-hand side of (5.22) is now absent. This in turn gives $m^2=-5$, which leads to a scalar field in a non-unitary representation (complex E). This algebraic construction is therefore pathological.

Each independent (traceless) structure $\Phi^{(p;0)}(n)$ gives rise to an infinite expansion in terms of even powers of K, such that Φ can be written in terms of elements of the form

$$f\left(n;K^2\right)\Phi^{(p;0)}(n)\,,\qquad n=p+m\,,\qquad m=0,2,\dots\,, \tag{4.23}$$

where the analytic function $f(n;z)$ is defined by

$$f(n;z)=\sum_{k=0}^{\infty}\frac{z^k}{k!\left(k+n+\frac{3}{2}\right)_k}\,. \tag{4.24}$$

For example, the scalar field is represented by the expansion:

$$\left(1+\frac{2}{5}K^2+\frac{2}{35}K^4+\frac{4}{945}K^3+\cdots\right)\phi\,, \tag{4.25}$$

where ϕ is the y and $\bar{y}$ independent component of Φ.

5. Linearized constraints

The first step towards finding the full field equations for the higher spin gauge theory based on the hs(2,2) algebra is to identify the appropriate linearized field equations. The requisite for writing these are the hs(2,2) covariant curvature and scalar master fields defined in the previous section. The basic assumption is that the higher spin gauge theory should make sense as an expansion around the AdS vacuum described by

$$\Phi=0\,,\qquad [A]=[\Omega]\,, \tag{5.1}$$

where Ω is the "flat" AdS connection:[7]

$$\begin{aligned}\Omega_\mu&=i\left(e_\mu{}^aP_a+\frac{1}{2}\omega_\mu{}^{ab}M_{ab}\right)\,,\\ F_\Omega&=d\Omega+\Omega\star\Omega=i\left(T^aP_a+\frac{1}{2}\left(R^{ab}+e^a\wedge e^b\right)M_{ab}\right)=0\,.\end{aligned} \tag{5.2}$$

Here $e_\mu{}^a$ and $\omega_\mu{}^{ab}$ are the fünfbein and Lorentz connection and T^a and R^{ab} the torsion and Riemann curvature two-forms defined by:

$$T^a=de^a+\omega^a{}_b\wedge e^b\,,\qquad R^{ab}=d\omega^{ab}+\omega^a{}_c\wedge\omega^{cb}\,. \tag{5.3}$$

The resulting five-dimensional Einstein equation with cosmological constant Λ reads:

$$R_{\mu\nu}-\frac{1}{2}(R+\Lambda)g_{\mu\nu}=0\,,\qquad \Lambda=-\frac{12}{R^2}\,, \tag{5.4}$$

[7]We have chosen units such that the AdS radius $R_{\rm AdS}=1$. It can be introduced by replacing $P_a\to R_{\rm AdS}P_a$. The insertions of powers of $R_{\rm AdS}$ in the component formulae are then determined by dimensional analysis.

where the metric and the Ricci tensor have been defined by

$$g_{\mu\nu} = e_\mu{}^a e_{\nu a} \,, \qquad R_{\nu a} = e_b{}^\mu R_{\mu\nu,a}{}^b \,. \tag{5.5}$$

In the AdS vacuum we find

$$R_{\mu\nu}{}^{ab} = -2e_{[\mu}{}^a e_{\nu]}{}^b \,, \qquad R_{\mu\nu} = 4g_{\mu\nu} \,. \tag{5.6}$$

The normalization is such that the AdS metric is given by

$$ds^2 = \frac{1}{r^2}\left(dr^2 + dx^2\right) \,, \tag{5.7}$$

in five-dimensional Poincaré coordinates.

Assuming that the full equations have a curvature expansion in powers of Φ, a linearization of these curvatures around the AdS background should give rise to free equations describing the massless degrees of freedom in the spectrum $\mathcal{S}$ defined in (3.22), that is, the free equations for massless fields of spin $s = 0, 2, 4, \ldots$ and AdS energy $E = s + 2$. In the case of $s = 0$ this corresponds to the Klein-Gordon equation:[8]

$$(\nabla^\mu \partial_\mu + 4)\phi = 0 \,, \tag{5.8}$$

where ϕ is an independent scalar (which will turn out to be the leading component of the master scalar field). The linearized spin two equation can of course be obtained by linearizing (5.4). However, the formalism that appears to be the most convenient in the context of higher spin gauge theory is a generalization of the first order constraint formulation of (5.4). In the spin two case this amounts to solving for the auxiliary Lorentz connection in terms of the dynamical fünfbein from the torsion constraint $T^a = 0$, and writing the Einstein equation as a constraint on the AdS covariantization $F^{ab} = R^{ab} + e^a \wedge e^b$ of the Riemann curvature. This tensor contains 50 components in five dimensions, of which 15 are set equal to zero by the Einstein equation. The remaining 35 non-vanishing components define the spin two Weyl tensor. It corresponds to the Young tableaux (4.14) with $m = 2$ and $p = 0$ (the "window" diagram), or equivalently a multi-spinor with Young tableaux (4.12) defining a totally symmetric multi-spinor $\Phi^{(0;0)}_{\alpha\beta\gamma\delta}$. This choice of m and p does not correspond to an algebra element; the Weyl tensor is obtained by converting both the algebra-valued tangent space indices ab and the curved indices $\mu\nu$ on the spin two curvature $F_{\mu\nu,ab}$ into spinor indices by using the fünfbein and Dirac matrices.

[8]Using (5.7) and making the ansatz $\phi \sim r^E$ we find that the Klein-Gordon equation $(\nabla^\mu\nabla_\mu - m^2)\phi = 0$, which follows from the usual free action $\frac{1}{2}\int d^5x\sqrt{-g}(\partial^\mu\phi\partial_\mu\phi - m^2\phi^2)$ with "positive" m^2, leads to the characteristic equation $E(E-4) = m^2$, where E and m^2 is given in units of $R_{\rm AdS}$. For $E = 2$ we find $m^2 = -4$, which saturates the lower bound for m^2 (see, for example, [41]).

Thus, in this language the (full) Einstein equation with cosmological constant (5.4) can be written as the following constraint:

$$F_{\mu\nu,ab} = (\Gamma_{ab})^{\alpha\beta}(\Gamma_{cd})^{\gamma\delta} e_\mu{}^c e_\nu{}^d \Phi^{(0;0)}_{\alpha\beta\gamma\delta}\,, \qquad T_{\mu\nu,a} = 0\,. \tag{5.9}$$

The higher spin generalization of these curvature constraints has been given in the free case in a linearization around the AdS vacuum in [32, 33] using a tensorial basis with lorentzian indices. These constraints are straightforward to cast into the spinorial basis. The higher spin dynamics also requires a constraint on the scalar master field Φ. Since it is already linear in fluctuations, the only possible constraint linearized constraint on Φ is the vanishing of $D_\Omega\Phi$. Using the notation of (4.12) the linearized constraints therefore read ($n = 2\ell + 1$)

$$F_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n} = e^a \wedge e^b (\Gamma_{ab})^{\gamma\delta} \Phi^{(0;0)}_{\gamma\alpha_1\cdots\alpha_n,\delta\beta_1\cdots\beta_n}\,, \tag{5.10}$$

$$d\Phi + \Omega \star \Phi - \Phi \star \pi(\Omega) = 0\,, \tag{5.11}$$

where F is the linearized curvature

$$F = dA + \Omega \star A + A \star \Omega\,. \tag{5.12}$$

The left-hand side of (5.10) contains all possible spinorial index structures compatible with the fact that F is an element of the ℓth level of $\mathcal{G}^{(0)}$, while the right-hand side only contains the symmetric spin $s = 2\ell+2$ tensor $\Phi^{(0;0)}(2s)$ (without K^2-expansion), which is the higher spin generalization of the spin two Weyl tensor $\Phi^{(0;0)}_{\alpha\beta\gamma\delta}$. Thus (5.10) contains generalized torsion constraints, field equations as well as the identification of the generalized Weyl tensors.

We remark that whereas the constraint (5.11) on the master scalar field is written in terms of functions of y and $\bar{y}$, the constraint (5.10) on the curvature has been written in component form. The reason for this is that whereas the full constraint on the master scalar field has to be of the form $D_A\Phi = \mathcal{V}_1(A, \Phi)$, where $\mathcal{V}_1$ is linear[9] in A and quadratic in Φ, so that its linearization is given uniquely by (5.11), the full form of the curvature constraint (5.10) is $F_A = \mathcal{V}_2(A; \Phi)$ for some function $\mathcal{V}_2$ which is quadratic in A and linear in Φ. Thus the implication of (5.10) is that whatever form $\mathcal{V}_2$ has, its linearization around the AdS vacuum must be given by the right-hand side of (5.10). Some further remarks on the curvature expansion of the full theory are given in the conclusions.

The constraints (5.10) and (5.11) are integrable. The integrability of (5.11) follows from the flatness of Ω, as given in (5.2). The integrability of (5.10) requires the Bianchi identity

$$dF + \Omega \star F - F \star \Omega = 0 \tag{5.13}$$

[9]It is not obvious that $\mathcal{V}_1$ cannot depend on $d\Phi$; in fact, in four dimensions this fact was shown only recently in [28]. We expect that the same holds in five dimensions.

to be satisfied when F is substituted using (5.10). To examine this equation we write the constraints (5.10) and (5.11) in component form:

$$\begin{aligned} F_{\mu\nu,\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n} &\equiv 2\nabla_{[\mu}A_{\nu],\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n} + \\ &\quad + n\left(\left(\Gamma_{[\mu}\right)_{(\alpha_1|}{}^{\gamma}A_{\nu],\gamma|\alpha_2\cdots\alpha_n),\beta_1\cdots\beta_n} - \left(\Gamma_{[\mu}\right)_{(\beta_1|}{}^{\gamma}A_{\nu],\alpha_1\cdots\alpha_n,\gamma|\beta_2\cdots\beta_n)}\right) \\ &= \frac{1}{8}(\Gamma_{\mu\nu})^{\gamma\delta}\Phi^{(0;0)}_{\gamma\alpha_1\cdots\alpha_n,\delta\beta_1\cdots\beta_n}\,, \end{aligned} \tag{5.14}$$

$$\nabla_\mu\Phi_{\alpha_1\cdots\alpha_n,}{}^{\beta_1\cdots\beta_n} - \frac{1}{2}(\Gamma_\mu)^{\gamma\delta}\Phi_{\gamma\alpha_1\cdots\alpha_n,\delta}{}^{\beta_1\cdots\beta_n} + \frac{n^2}{2}(\Gamma_\mu)_{(\alpha_1}{}^{(\beta_1}\Phi_{\alpha_2\cdots\alpha_n),}{}^{\beta_2\cdots\beta_n)} = 0\,. \tag{5.15}$$

Here ∇_μ is the Lorentz covariant derivative. We note that the multi-spinors in the last equation are the coefficients of the y and $\bar{y}$ expansion of the master scalar field including the K^2-expansions (4.23). The component form of the Bianchi identity (5.13) reads:

$$\begin{aligned} \nabla_{[\mu}F_{\nu\rho],\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_n} + \frac{n}{2}\Big(\left(\Gamma_{[\mu}\right)_{(\alpha_1|}{}^{\gamma}F_{\nu\rho],\gamma|\alpha_2\cdots\alpha_n),\beta_1\cdots\beta_n} - \\ - \left(\Gamma_{[\mu}\right)_{(\beta_1|}{}^{\gamma}F_{\nu\rho],\alpha_1\cdots\alpha_n,\gamma|\beta_2\cdots\beta_n)}\Big) = 0\,. \end{aligned} \tag{5.16}$$

By inserting (5.14) in (5.13) and making use of (5.15) to substitute for $\nabla_\mu\Phi^{(0;0)}$ by

$$\nabla_\mu\Phi^{(0;0)}_{\alpha_1\cdots\alpha_m,\beta_1\cdots\beta_m} = \frac{1}{2}(\Gamma_\mu)^{\gamma\delta}\Phi^{(1;0)}_{\gamma\alpha_1\cdots\alpha_m,\delta\beta_1\cdots\beta_m}\,, \tag{5.17}$$

(the K-expansion of Φ does not affect this sector), it follows that (5.13) holds due to the following Fierz identities:[10]

$$\left(\Gamma_{a[b}\right)^{\alpha\beta}\left(\Gamma_{cd]}\right)^{\gamma\delta}\Phi^{(0;0)}_{\alpha\beta\gamma\delta\cdots} = 0\,, \tag{5.19}$$

$$\left(\Gamma_{[b}\right)^{\alpha\beta}\left(\Gamma_{cd]}\right)^{\gamma\delta}\Phi^{(1;0)}_{\alpha\beta\gamma\delta\cdots} = 0\,, \tag{5.20}$$

The symmetries and the tracelessness of the multi-spinors contracting the Dirac matrices are important for these identities to be satisfied.

Using the equivalence between the spinorial and tensorial bases established in section 4 it is straightforward to see that the constraint (5.10) is equivalent to the curvature constraints which were shown in [32, 33] to give rise to a massless spin s degree of freedom. Thus (5.10) sets all components of the curvature except the generalized Weyl tensors equal to zero. The vanishing curvatures are the generalized torsion equations and the spin $s \geq 2$ field equations ($s = 2\ell + 2 = 2, 4, \ldots$). The torsion equations are algebraic equations for the auxiliary gauge fields $A^{(p;0)}_\mu(2s-2)$, $0 \leq p \leq s-2$, which can be solved in terms of the generalized fünfbeins $A^{(s-1;0)}_\mu(2s-2)$. The remaining vanishing curvatures then become second-order field equations,

[10]The spinor conventions given in section 2 are such that the following Fierz identity holds:

$$M^{\alpha\beta}N^{\gamma\delta} = -\frac{1}{8}\left(M\Gamma^{ab}N\right)^{\alpha\delta}(\Gamma_{ab})^{\beta\gamma} - \frac{1}{4}(M\Gamma^aN)^{\alpha\delta}(\Gamma_a)^{\beta\gamma} - \frac{1}{4}(MN)^{\alpha\delta}C^{\beta\gamma}\,. \tag{5.18}$$

which after gauge fixing give rise to mode expansions based on the massless SO$(4,2)$ weight spaces $D(s,s;s+2)$. Thus the gauge fields give rise to the spin $s\geq 2$ sector of the spectrum (3.22).

The non-vanishing curvature components in (5.10) are those corresponding to the SU$(2,2)$ Young tableaux (4.12) with $m=2\ell+2$, $p=0$, that is the SO$(4,1)$ Young tableaux (4.14) with $n_1=n_2=m$. These are the generalized Weyl tensors $\Phi^{(0;0)}(m)$, which are totally symmetric multi-spinors. From the constraint (5.11), which is written in components in (5.15), it follows that the trajectory $\Phi^{(p;0)}(m+p)$ ($p=0,1,2,\ldots$) with fixed $m=0,2,\ldots$ corresponds to the derivatives of the leading tensor $\Phi^{(0;0)}(m)$. Hence the only independent component of the scalar master field is the single real scalar field $\phi\equiv\Phi^{(0;0)}(0)$. From (5.15) it follows that

$$\partial_\mu\phi = \frac{1}{2}(\Gamma_\mu)^{\alpha\beta}\Phi^{(1;0)}_{\alpha,\beta}\,, \tag{5.21}$$

$$\nabla_\mu\Phi^{(1;0)}_{\alpha,\beta} = \frac{1}{2}(\Gamma_\mu)^{\gamma\delta}\left[\Phi^{(2;0)}_{\alpha\beta,\gamma\delta}+\frac{2}{5}C_{(\alpha|\beta}C_{|\gamma)\delta}\phi\right]-\frac{1}{2}(\Gamma_\mu)_{\alpha\beta}\phi\,, \tag{5.22}$$

where we use the notation of (4.16). The trace part in the first term on the right-hand side of (5.22) comes from the K^2-expansion of the scalar field according to (4.25). This term is necessary for obtaining the scalar field equation with the critical mass term that is appropriate for its being AdS-massless, and hence the importance of the condition (4.19). Indeed, combining the two equations given above and making use of the Fierz identity

$$(\Gamma^a)^{\alpha\beta}(\Gamma_a)^{\gamma\delta}\Phi^{(2;0)}_{\alpha\beta\gamma\delta}=0\,, \tag{5.23}$$

we find that the scalar field satisfies the scalar field eq. (5.8), which gives rise to a mode expansion based on the spin zero weight space $D(0,0;2)$ in the spectrum (3.22).

6. Summary and remarks

We have used Grassmann even spinor oscillators to construct a bosonic higher spin extension hs$(2,2)$ of the five-dimensional AdS algebra SU$(2,2)$ containing generators giving rise to dynamical as well as auxiliary gauge fields with spins $s=2,4,6,\ldots$ upon gauging. The higher spin algebra is naturally embedded into a larger algebra $\mathcal{G}$ as the coset $\mathcal{G}/\mathcal{I}$ where $\mathcal{I}$ is an ideal of $\mathcal{G}$ generated by arbitrary $\star$-polynomials of the central element K multiplied by traceless polynomials of y and $\bar{y}$ (see (3.14) and (3.16)). The large algebra $\mathcal{G}$ can be represented unitarily and irreducibly on a spin j doubleton with $|K|=j/2$, while the higher spin algebra hs$(2,2)$ has a well-defined representation only on the spin zero doubleton with $K=0$. The symmetric tensor product of two spin zero doubletons gives rise to a unitary irreducible representation $\mathcal{S}$ of hs$(2,2)$ that decomposes into spin $s=0,2,4,\ldots$ weight spaces of SO$(4,2)$.

We expect $\mathcal{S}$ to be the spectrum of a five-dimensional gauge theory with local $\mathrm{hs}(2,2)$ symmetry and an AdS vacuum with unbroken global $\mathrm{hs}(2,2)$ symmetry. As a first step towards constructing this theory, we have shown that the spin s gauge fields which arises upon gauging $\mathrm{hs}(2,2)$ are in one-to-one correspondence with the set of spin s gauge fields which were used in [32, 33] to construct linearized curvature constraints describing a massless spin s field in five dimensions. We have converted these constraints, which were originally given in the lorentzian basis, into the spinor basis, where they can be written as (5.10). Furthermore have identified a representation for the scalar master field which contains the physical spin zero field, the generalized higher spin Weyl tensors and their derivatives. The scalar master constraint is simply given by the vanishing its AdS covariant derivative. We remark that while the expression (5.12) of the linearized curvature is equivalent to the one used in the formulation of [32, 33], their formulation neither incorporates the oscillator algebra (3.1) and (3.8) required for constructing the expressions (4.6) and (4.7) for the non-linear curvature and covariant derivative, nor the master scalar field ϕ constructed in section 4, which plays a crucial role in higher spin gauge theory.

In terms of the oscillator y_α and its Majorana conjugate $\bar{y}_\alpha$ the $\mathrm{U}(2,2)$ subalgebra is spanned by the bilinears $y_\alpha \bar{y}_\beta$. The remaining generators of $\mathcal{G}$ form levels labeled by an integer ℓ, such that $\mathrm{U}(2,2)$ is the zeroth level and the ℓth level is spanned by monomials that contain $2\ell+1$ y_α-oscillators and the same number of $\bar{y}_\alpha$-oscillators. The algebra elements may be written either using the Weyl-ordered (fully symmetrized) oscillator product as in (3.9), or by extracting explicitly positive powers of $K\star$ as in (3.14). These correspond to traces taken using the anti-symmetric charge conjugation matrix $C_{\alpha\beta}$. In the latter basis, the ideal $\mathcal{I}$ is given by the space of arbitrary polynomials containing a strictly positive number of $K\star$ factors. Thus $\mathrm{hs}(2,2)$ is isomorphic to a space of traceless and real multispinors that is arranged into levels such that the elements in the ℓth level carry two sets of $2\ell+1$ symmetrized spinor indices (one set contracted with y's and the other set contracted with $\bar{y}$'s). The ℓth level can be decomposed further by anti-symmetrizing p pairs of spinor indices taken from the two sets ($0 \le p \le 2\ell+1$) and symmetrizing the remaining indices, as represented by the Young tableaux (4.12). For given p these are in one-to-one correspondence with the Lorentz-tensor represented by a two-row Young tableaux with $2\ell+1$ boxes in the first row and $2\ell+1-p$ in the second, as given in (4.14).

Upon gauging, we thus find the gauge field content required for writing the above-mentioned curvature constraints. The gauge fields corresponding to the generators with $p = 0, \dots, 2\ell$ are auxiliary, while $p = 2\ell+1$ corresponds to the dynamical gauge field $A_{\mu,a_1,\dots,a_{2\ell+1}}$. Of particular importance is also the curvature corresponding to $p = 0$, that is $R_{\mu\nu,a_1b_1,\dots,a_{2\ell+1}b_{2\ell+1}}$ where each pair a_ib_i is anti-symmetric. This curvature contains the only spin $2\ell+2$ curvature components that are non-vanishing on-shell. These define the generalized spin $2\ell+1$ Weyl tensor, which is a fully symmetric, real and traceless multispinor with $2(2\ell+2)$ spinor indices occurring on the right-hand side of the curvature constraint (5.10).

Whereas the gauge fields fit naturally into the adjoint representation of $hs(2,2)$, perhaps a less obvious issue in the construction is to determine which $hs(2,2)$ representation contains the Weyl tensors. To this end, we first observe that although the Weyl tensors have spins $2,4,\dots$ it is natural to fit them into a scalar master field. This is because the constraint algebra is written as a free differential algebra, or a Cartan integrable system, which means that for each p-form with $p>0$ there will be a corresponding $(p-1)$-form gauge parameter (the spacetime diffeomorphism group is automatically incorporated into the gauge group such that a vector field corresponds to field dependent $(p-1)$-form gauge parameters given by the inner derivatives of the corresponding p-form potentials). Moreover, since the spectrum $\mathcal{S}$ contains a spin zero degree of freedom, it is natural to attempt to unify the corresponding scalar field with the Weyl tensors in a scalar master field.

This stage of the construction reveals an intimate interplay between the group theoretical constraints and the dynamics. The $hs(2,2)$ transformation property (4.5) of the scalar master field is determined by the requirement that it should contain the Weyl tensors and the scalar field, which amounts to the constraint (4.2) involving the involution π. This in turn determines the form of its gauge covariant derivative (4.7), where we in particular note the twisting of the connection in the last term by the insertion of π. At the linearized level the only natural, gauge invariant constraint on the scalar master field, which we treat as a linear fluctuation around a zero background value, is to set its background covariant derivative to zero. This turns out to yield the correct scalar equation as well as constraints on the remaining components of the scalar master field which are consistent with identifying the fully symmetric higher spin multispinors with the Weyl tensors (the latter amounts to verifying the Fierz identities (5.19) and (5.20)). The twisting, which flips the sign of the fünfbein contribution while it keeps the sign of the contribution from the Lorentz connection, that is $\pi(P_a)=-P_a$ and $\pi(M_{ab})=M_{ab}$, plays a crucial role in all this. In fact, if one takes a scalar master field in the adjoint representation and set its adjoint covariant derivative to zero, then it will be constant.

It is interesting to note that the definition of π in five dimensions relies on the fact that the Dirac matrices with one and two vector indices have different symmetry properties. On the other hand, these Dirac matrices have the same symmetry seven dimensions. Thus the five-dimensional π cannot be generalized to seven dimensions. The problem of finding appropriate twist-operations in higher dimensions has been studied further in [36].

Clearly, the analysis in this paper only contains the first step towards building a full higher spin gauge theory in five dimensions, and it still remains to construct the interactions. To this end, we believe that the results of this paper provide the correct framework for building the interactions. Moreover, experience with the higher spin gauge theory in $D=4$ suggests an efficient method for gauging based on the spinorial formulation presented here, consisting of embedding of the full, non-linear constraint algebra into an enlarged constraint algebra based on an extension of the ordinary spacetime by an auxiliary non-commutative spinorial Z-space a lá Vasiliev [23]. Indeed suggestions for how this might be done in the case of even spacetime dimension has already been given quite some time ago [31]. We are currently investigating constructions of similar type in the case of five dimensions, though our results are not conclusive at this point mainly due to problems with identifying the proper constraint on the master curvature in the extended space.

The spinorial oscillators are also useful in constructing supersymmetric extensions. The higher spin extension $\mathrm{hs}(2,2|n)$ of the finite-dimensional supergroup $SU(2,2|n)$ containing the bosonic subgroup $SO(4,2)\times SU(n)$ and odd supercharges Q^i_α, $i=1,\ldots,n$, can then be constructed by introducing an additional set of Grassmann odd complex oscillators θ^i forming a Clifford algebra, and setting $Q^i_\alpha=y_\alpha\theta^i$. The oscillator realization of $SU(2,2|n)$ contains the generator $Z=K+\frac{1}{2}\theta^i\theta_i$ which becomes central in the higher spin superalgebra [42]. Similar constructions in $D=2n$ involving Kleinian operators have been suggested in [31]). We expect the spectrum to be generated by the CPT self-conjugate superdoubleton, which has vanishing Z. Indeed, a preliminary analysis indicates that the results of this paper will generalize in a rather straightforward fashion to the maximal case $\mathrm{hs}(2,2|4)$. The main subtlety resides in the fact that the scalar master field contains not just the gauge matter sector, but also the spin one three-form field strength and a tower of higher spin generalizations thereof.

As pointed out recently by [29, 30] the five-dimensional sphere compactification of type-IIB string theory with $N\geq 1$ units of RR five-form flux and zero string coupling should have a description in terms of a five-dimensional theory governed by massless higher spin gauge invariance, which is dual to free $U(N)$ Yang-Mills theory at the boundary, that is, the theory of N^2 spin one conformal superdoubletons. Zero string/gauge coupling implies infinite string length (the ratio of the string length to the radius diverges when the string coupling becomes small at fixed N), which means tensionless strings. The theory is thus parameterized by the five-dimensional Planck scale ℓ_p and the AdS radius $R=N^{1/3}\ell_p$. The boundary correlation functions can be constructed from the basic single-trace operators. In particular the bilinear single-trace operators give rise to currents that couple to massless AdS modes. The spectrum of such currents is isomorphic to the product of two superdoubletons, i.e. the massless spectrum of the above-mentioned $\mathrm{hs}(2,2|4)$ gauge theory.

In the field theory limit $\ell_p\ll R$, that is the large-N limit, the $\mathrm{hs}(2,2|4)$ higher spin gauge theory has a well-defined curvature expansion [22, 28] at energies corresponding to length scales ℓ in the interval $\ell_p\ll\ell\ll R$. The above-mentioned duality therefore implies that this expansion is dual to the interactions of the bilinear currents. Hence this setup offers a parameter regime in which the strong version of the Maldacena conjecture can actually be tested directly! Importantly, even though the 't Hooft coupling vanishes, so that correlation functions where all operators are single (linear) doubleton fields are free, the correlation functions involving the current-bilinears have a non-trivial generating functional, which should be equal to the effective action of the $hs(2,2|4)$ gauge theory. Since these interactions persist at zero string coupling they may be considered to be the basic "M-interactions" defining M-theory in an unbroken phase.

The boundary theory also contains operators in the form of normal-ordered products of three or more doubletons. These correspond to massive bulk modes, which form massive higher spin multiplets. It would be interesting to investigate whether the full bulk spectrum originates from a massless higher spin gauge theory in ten dimensions. Indeed each higher spin multiplet (the massive ones as well as the massless one) contain a CPT self-conjugate spin two multiplet. These give rise to a tower of spin two multiplets describing the Kaluza-Klein modes of the ten-dimensional supergravity multiplet.

The addition of Yang-Mills interactions break the higher spin currents in four dimensions [43]. This implies to that the bulk theory has a finite string coupling, that is, a finite string mass. In this massive phase we expect some of the higher spin gauge symmetries to be realized as Stückelberg-like shift symmetries, with a smooth limit (in the sense that there is no jump in degrees of freedom) to higher spin gauge symmetry as the mass-parameter is sent to zero. A better understanding of this may cast light on the nature of perturbative string theory in AdS backgrounds as well as on the issue of how to incorporate the massive multiplets, as some of these may have to be included in the perturbative spectrum in order for the Higgs mechanism to work consistently.

We expect the bosonic theory considered in this paper to be a consistent truncation of the $N = 4$ supersymmetric case as follows. In the boundary we set all fields in a given superdoubleton multiplet equal to zero except one of the scalars. In the bulk, the spectrum of the supersymmetric theory consists of a tower of supermultiplets arranged into levels $\ell = 0, 1, 2, \ldots$. In the truncation to our model, we keep the graviton of the supergravity multiplet at $\ell = 0$, the spin zero and spin four fields at $\ell = 1$ and the field with maximal spin $s_{\rm max} = 2\ell + 2$ at level ℓ. Effectively this amounts to setting $\theta^i = 0$ in the notation introduced above. The bosonic model may therefore serve as a simplified setup for addressing some of the above issues, such as the couplings to massive multiplets and the ten-dimensional origin. The bulk dilaton is thrown away in this truncation, however, which leaves in doubt whether it may facilitate the massive string deformation. Further evidence against this is that the deformation of the boundary scalar theory by adding a ϕ^4 coupling, which is analogous to the introduction of finite $g^2_{\rm YM}$ in the super case, breaks the conformal invariance at the quantum level.

In [26] it was conjectured that the seven-sphere compactification of M-theory with N units of four-form flux leads to a duality in four spacetime dimensions, which is similar to the one discussed above in five dimensions. Here the free three-dimensional singleton is dual to the strong coupling limit, that is $\ell_p \sim R$, of the four-dimensional higher spin theory with gauge group shs(8|4). At weak coupling, that is $\ell_p \ll R$, it has a curvature expansion for energies corresponding to length scales $\ell_p \ll \ell \ll R$, which is expected to be dual to the mysterious theory of $N \gg 1$ coinciding membranes (and analogously coinciding five-branes are expected to be dual to weakly coupled higher spin theory in seven dimensions).

In fact, from the higher spin point of view there appears to be a parity between the IIB and the IIA/11D corners of M-theory, in the sense that both give rise to similar higher spin gauge theories. The differences due the presence of string coupling in IIB and the absence thereof in $D = 11$ instead seems to reside with the patterns of symmetry breaking, which appears to be a property of the gauged supergravities rather than the full higher spin theory. Thus, it is tempting to think of an unbroken M-gauge theory embracing both IIB and 11D in a unified framework. We will elaborate further on this theme in a separate publication [42].

To conclude, we believe that higher spin gauge theories do fit naturally into the M-theory jig-saw and that they will eventually provide new and fascinating insights to hitherto uncharted limits of M-theory.

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